

國立新竹教育大學

103 學年度碩、博士班(含碩士在職專班)招生考試試題

系所別：應用數學系碩士班

科目：微積分 (總分 150 分)

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※請橫書作答

1. Find the following limit (if it exists). If it does not exist, explain why. (20%)

(a) $\lim_{x \rightarrow \infty} \frac{\sqrt[3]{1} + \sqrt[3]{2} + \cdots + \sqrt[3]{n}}{n^{4/3}}$. (b) $\lim_{x \rightarrow 0} [x+1]$, where $[x]$ = greatest integer n such that $n \leq x$.

(c) $\lim_{x \rightarrow 0} \frac{\sin 5x}{3x}$. (d) $\lim_{x \rightarrow -\infty} (3x + \sqrt{9x^2 + 16})$.

2. Evaluate the following integrals. (25%)

(a) $\int 16x^3 \sqrt{3x^4 + 2} dx$. (b) $\int 2 \sin^3 x \cos x dx$.

(c) $\int_0^1 \frac{3e^{3x}}{1+e^{3x}} dx$. (d) $\int_0^\pi e^{2 \sin x} \cos x dx$

(e) $\int x \sin x + xe^x dx$.

3. Find the derivatives $F'(x)$. (10%)

(a) $F(x) = \int_1^{\cos x} t^4 dt$. (b) $F(x) = \ln x + \tan x$.

4. Prove that if $a > 0$ and n is any positive integer, then the polynomial function

$p(x) = x^{2n^2+1} + ax^3$ cannot have two real roots. (10%)

5. Find the volume of the solid of revolution formed by revolving the region bounded by

$y = x - x^3$

and the x -axis ($0 \leq x \leq 1$) about the y -axis. (10%)

6. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2}{9n^2 - 3n - 2}$. (10%)

7. How large must n be so that the partial sum $\sum_{k=1}^n \frac{1}{k^3 + 1}$ approximates $\sum_{k=1}^{\infty} \frac{1}{k^3 + 1}$ with an error of no more than 0.000002. (15%)

8. Find the convergence set for $\sum_{n=3}^{\infty} \frac{x^n \ln n}{n4^{2n}}$. (15%)

9. Consider the polar equation $r = 4 \sin 3\theta$, find the area of one leaf. (15%)

10. Evaluate $\iint_D xy \, dA$, where D is the region bounded by the line $y = -x + 6$ and parabola $y = 3x^2 + 2$. (10%)

11. Find the equation of tangent plane to the sphere $x^2 + y^2 + z^2 = 25$ at the point $(1, 1, \sqrt{3})$. (10%)