

國立新竹教育大學九十九學年度研究所碩士班招生考試試題

所別：應用數學系碩士班

科目：微積分（含初等微積分、高等微積分）（本科總分：150分）

※請橫書作答

1. (21%) In the following, find $\frac{dy}{dx}$

(a) $4\cos x \sin y = 1$ (b) $y = \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2$ (c) $y = \int_{\sqrt{x}}^{x^3} \sqrt{t} \sin t dt$

2. (28%) Evaluate the following integral

(a) $\int \frac{x^2}{\sqrt{1-x}} dx$ (b) $\int_0^{\pi/2} \sin^2 x \cos^2 x dx$ (c) $\int \frac{1}{\sqrt{x^2+16}} dx$ (d) $\int e^x \sin x dx$

3. (8%) Find the limit if it exists or show that the limit does not exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{6x^3y}{2x^4 + y^4}$$

4. (8%) Find the shortest distance from the point $(1, 0, -2)$ to the plane $x+2y+z=4$.

5. (10%) The region D enclosed by the curves $y^2 = x$ and $x = 2y$ is rotated about the y-axis. Find the volume of the resulting solid.

6. (28%) Examine the following series for convergence:

(a) $\sum_{k=1}^{\infty} k e^{-k^2}$, (b) $\sum_{k=1}^{\infty} \left(\frac{k+1}{k^2+1}\right)^3$, (c) $\sum_{n=1}^{\infty} (\sqrt{1+n^2} - n)$, (d) $\sum_{n=1}^{\infty} n^3 e^{-n}$.

7. (12%) Prove that there is a solution of the equation

$$\frac{1}{\sqrt{x+x^2}} + x^2 - 2x = 0, \quad x > 0.$$

8. (15%) If f has a finite third derivative f''' in $[a, b]$ and if

$$f(a) = f'(a) = f(b) = f'(b) = 0.$$

Prove that $f'''(c) = 0$ for some c in (a, b) .

9. (20%) Discuss whether the following sets are open or closed. Justify your answers.

(a) $S = \{(x, y) \in \mathbb{R}^2 \mid x \text{ is rational}\}$. (b) $S = \{(x, y) \in \mathbb{R}^2 \mid xy > 1\}$.

(c) $S = \{(-1)^n + 1/m \mid m, n \in \mathbb{N}\}$. (d) $S = \{2^{-n} + 5^{-m} \mid m, n \in \mathbb{N}\}$.